Module 1, Add on math lesson – Simultaneous Equations

Student

45 minutes

Purpose of this lesson

- Practice solving simultaneous equations.
- See how simultaneous equations are a useful tool for solving real world problems.
- Prove the potential divider equation used in Module 1, core lesson 4.

Analyzing electric circuits

Electricity involves electrons flowing through the wires and components of a circuit. Electric current (denoted **I**) is equivalent to the amount of water flowing through a pipe (e.g. volume of water per second). Voltage (denoted **V**) is equivalent to the pressure which forces the water through the pipe.

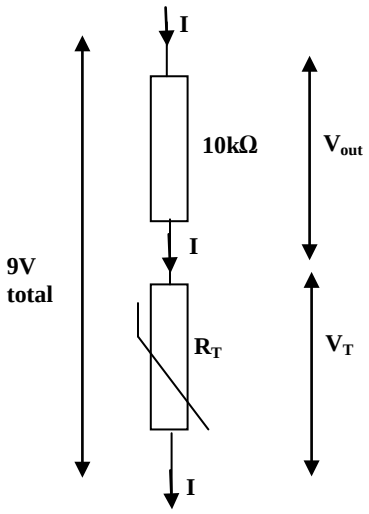

Fig. 1. Thermistor (bottom) and resistor (top) form a *potential divider* circuit. Due to the battery, there is 9V across the two resistors. This total 9V is *divided* between the resistor and the thermistor. Some of it, V_{out}, is applied across the resistor and the remainder V_T is applied across the thermistor.

Look at the simplified circuit diagram, fig. 1. Any current which passes through the 10kΩ resistor must also pass through the thermistor (think about water flowing through two pipes connected end-to-end).

Note that:

- The same current, **I**, passes through both resistors.
- The total voltage across both resistors is 9V due to the battery.
- This voltage is divided between the 10kΩ resistor and the thermistor, whose resistance, $\mathbf{R_T}$, varies with temperature.
- Using a multimeter, we can measure the portion of the voltage across the 10kΩ resistor. This will be our sensor's "output signal", so we have labeled it $\mathbf{V_{out}}$.
- Note 10kΩ means "ten thousand Ohms".

This sensor circuit is designed to measure temperature. Therefore, to analyze the sensor, derive a formula which relates the sensor's output signal, $\mathbf{V_{out}}$, to the quantity which varies with temperature, $\mathbf{R_T}$.

Analyze this circuit using Ohm's law, which states that for any resistor, **R**, the voltage, **V**, and current, **I**, are related by the equation **V=IR**. Apply Ohm's law twice to form two simultaneous equations.

Approach 1

The total battery voltage (9V) acts across the combination of two resistors (the resistances add) causing an unknown current, **I**, to flow through both.

Hence, Ohm's law	V	=	I	×	R	
Gives	9	=	I	×	(10000 + R_T)	**equation 1**

The measured voltage, $\mathbf{V_{out}}$, acts across the 10kΩ resistor alone, causing the same current, **I**, to flow through it.

Hence	V	=	I	×	R	
Gives	V_{out}	=	I	×	10000	**equation 2**

1a) Look at the temperature-resistance data collected in *Lesson 3 – Temperature vs. resistance characteristics of a thermistor*. For each temperature that you experimented with, substitute the value of thermistor resistance, $\mathbf{R_T}$, into equation 1 above. Now solve the pair of simultaneous equations (equation 1 and equation 2) to find the corresponding value of output voltage, $\mathbf{V_{out}}$. Hint: use the method of substitution or another method to eliminate **I**.

Temperature / °C	0	10	20	30	40	50	60	etc
$\mathbf{R_T}$ (look up from Lesson 3)								
$\mathbf{V_{out}}$ (find by solving equations)								

1b) Plot a graph of Temperature versus $\mathbf{V_{out}}$, with temperature on the vertical axis (y-axis) and $\mathbf{V_{out}}$ on the horizontal axis (x-axis). Compare this with similar data that you have measured or calculated in lessons 4 and 5.

More advanced:

1c) In part i) you solved the pair of simultaneous equations many times, using a different value for $\mathbf{R_T}$ each time, to find values of $\mathbf{V_{out}}$. Now see if you can find a general formula for $\mathbf{V_{out}}$ in terms of $\mathbf{R_T}$. Hint, keep $\mathbf{R_T}$ as a letter, but treat it the same way you did when you assigned it numerical values in part i) above.

Approach 2

Look at figure 1. The total voltage of 9Volts (from the battery) must be equal to the voltage across the thermistor plus the voltage across the 10kΩ resistor (we can call these $\mathbf{V_T}$ and $\mathbf{V_{out}}$).

i.e. $\mathbf{V_T} + \mathbf{V_{out}} = 9$

rearrange to get $\mathbf{V_T} = 9 - \mathbf{V_{out}}$

Now we can apply Ohms law to the thermistor:

Ohms law	V	=	I	×	R
Gives	$\mathbf{V_T}$	=	I	×	$\mathbf{R_T}$

Now combine the above two equations, by eliminating $\mathbf{V_T}$ (e.g. by method of substitution) to get:

	$(9 - \mathbf{V_{out}})$	=	I	×	$\mathbf{R_T}$ **equation 1**

Now apply Ohms law to the 10kΩ resistor:

Ohms law	V	=	I	×	R
Gives	$\mathbf{V_{out}}$	=	I	×	10000 **equation 2**

2a) Look at the temperature-resistance data collected in *Lesson 3 – Temperature vs. resistance characteristics of a thermistor*. For each temperature that you experimented with, substitute the value of thermistor resistance, $\mathbf{R_T}$, into equation 1 above. Now solve the pair of simultaneous equations (equation 1 and equation 2) to find the corresponding

value of output voltage, $\mathbf{V_{out}}$. Hint: use the method of substitution or another method to eliminate **I**.

Temperature / °C	0	10	20	30	40	50	60	etc
$\mathbf{R_T}$ (look up from Lesson 3)								
$\mathbf{V_{out}}$ (find by solving equations)								

2b) Plot a graph of Temperature versus $\mathbf{V_{out}}$, with temperature on the vertical axis (y-axis) and $\mathbf{V_{out}}$ on the horizontal axis (x-axis). Compare this with similar data that you have measured or calculated in lessons 4 and 5.

More advanced:

2c) In part i) you solved the pair of simultaneous equations many times, using a different value for $\mathbf{R_T}$ each time, to find values of $\mathbf{V_{out}}$. Now see if you can find a general formula for $\mathbf{V_{out}}$ in terms of $\mathbf{R_T}$. Hint, keep $\mathbf{R_T}$ as a letter, but treat it the same way you did when you assigned it numerical values in part i) above.

Problem for more advanced students

In *Lesson 3 – Temperature vs. resistance characteristics of a thermistor*, you observed that the resistance of your thermistor decreases with temperature according to a curved relationship.

i) What kind of relationship is this?

ii) Find an equation or formula that mathematically describes this decrease in resistance with temperature.

iii) Substitute this formula into equation 3 above, to find a mathematical expression which relates the output of your temperature sensor ($\mathbf{V_{out}}$) to temperature, **T**.